1. [5] Given right triangle ABC, with AB = 4, BC = 3, and CA = 5. Circle O passes through A and is tangent to BC at C. What is the radius of O?

2. [5] How many ways can you color the squares of a 2 x 2011 grid in 3 colors such that no two squares of the same color share an edge?

3. [5] A candy company makes 5 colors of jellybeans, which come in equal proportions. If I grab a random sample of 5 jellybeans, what is the probability that I get exactly 2 distinct colors?

4. [6] Simplify: i0 + i1 + … + i2011.

5. [6] How many perfect squares divide 23 \* 35 \* 57 \*79?

6. [6] An icosidodecahedron is a convex polyhedron with 20 triangular faces and 12 pentagonal faces. How many vertices does it have?

7. [6] Find the real solution(s) to the equation (x + y)2 = (x + 1)(y - 1).

8. [6] A sequence consists of the digits 122333444455555 . . . such that the each positive integer n is repeated n times, in increasing order. Find the sum of the 4501st and 4502nd digits of this sequence.

9. [6] There are three video game systems: the Paystation, the WHAT, and the ZBoz2π, and none of these systems will play games for the other systems. Uncle Riemann has three nephews: Bernoulli, Galois, and Dirac. Bernoulli owns a Paystation and a WHAT, Galois owns a WHAT and a ZBoz2π, and Dirac owns a ZBoz2π and a Paystation. A store sells 4 different games for the Paystation, 6 different games for the WHAT, and 10 different games for the ZBoz2π. Uncle Riemann does not understand the difference between the systems, so he walks into the store and buys 3 random games (not necessarily distinct) and randomly hands them to his nephews. What is the probability that each nephew receives a game he can play?

10. [7] Bob is writing a sequence of letters of the alphabet, each of which can be either uppercase or lowercase, according to the following two rules:

- If he had just written an uppercase letter, he can either write the same letter in lowercase after it, or the next letter of the alphabet in uppercase.

- If he had just written a lowercase letter, he can either write the same letter in uppercase after it, or the preceding letter of the alphabet in lowercase.

For instance, one such sequence is aAaABCDdcbBC. How many sequences of 32 letters can he write that start at (lowercase) a and end at (lowercase) z? (The alphabet contains 26 letters from a to z, without wrapping around, so that z does not precede a.)

11. [7] The product of the base seven numbers 247 and 307 is expressed in base seven. What is the base seven sum of the digits of this product?

12. [7] A student at Harvard named Kevin
Was counting his stones by 11
He messed up n times
And instead counted 9s
And wound up at 2007.
How many values of n could make this limerick true?
13. [8] The digits from 1 to 6 are arranged to form a six-digit multiple of 5. What is the probability that the number is greater than 500,000? Express your answer as a common fraction.

14. [8] The measures of the three angles of a triangle are in a ratio of 4:5:6. What is the measure in degrees of the greatest supplement of these three angles?

15. [8] In a game show, Bob is faced with 7 drawer, 2 of which hold prizes. Bob chooses a drawer. Then the host opens three other drawers, one of which has a prize. Bob chooses to switch to another drawer. What is the probability that his new drawer contains a prize?

16. [9] A spider is making a web between 12 distinct leaves which are equally spaced around a circle. He chooses a leaf to start at, and to make the base layer he travels to each leaf one at a time, making straight lines of silk from one leaf to another, such that no two of the lines of silk cross each other and he visits every leaf exactly once. In how many ways can the spider make the base layer of the web?